CBCS SCHEME

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17MAT11

First Semester B.E. Degree Examination, Jan./Feb. 2021

Engineering Mathematics – I

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Find the n^{th} derivative of $\frac{x}{(x-1)(2x+3)}$ **(06 Marks)**

b. Find the angle of intersection of the curves $r = \frac{a}{1+\cos\theta}$ and $r = \frac{b}{1-\cos\theta}$. **(07 Marks)**

c. Find the radius of curvature of the curve $x^4 + y^4 = 2$ at the point (1, 1). **(07 Marks)**

OR

2 a. If $y = e^{a\sin^{-1}x}$ prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+a^2)y_n = 0$. **(06 Marks)**

b. Find the pedal equation of $r^2 = a^2 \sec 2\theta$. **(07 Marks)**

c. Find the radius of curvature of the curve $r^n = a^n \sin n\theta$. **(07 Marks)**

Module-2

3 a. Obtain the Taylor's expansion of $\tan x$ about $x = \frac{\pi}{4}$ upto third degree terms. **(06 Marks)**

b. Evaluate $\underset{x\to 0}{\text{Lt}} \left(\frac{2^x+3^x+4^x}{3}\right)^{1/x}$ **(07 Marks)**

c. If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$ show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$ **(07 Marks)**

OR

4 a. If $u = \sin^{-1}\left(\frac{x^2y^2}{x+y}\right)$ then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3\tan u$. **(06 Marks)**

b. Obtain the Maclaurin's expansion of log(sec x) upto fourth degree terms. **(07 Marks)**

c. If $x + y + z = u$, $y + z = v$, $z = uvw$ find the Jacobian $J\left(\frac{x, y, z}{u, v, w}\right)$ **(07 Marks)**

Module-3

5 a. If $\phi = x^2 + y^2 + z^2$, $\vec{F} = x^2\hat{i} + y^2\hat{j} + z^2\hat{k}$ then find grad ϕ, div $\vec{F}$ and curl $\vec{F}$. **(06 Marks)**

b. A particle moves on the curve $x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$ where t is the time. Find the components of velocity and acceleration at $t = 1$ in the direction of the vector $i - 3j + 2k$. **(07 Marks)**

c. Prove that $\text{curl}(\phi\vec{A}) = \phi\,\text{curl}\vec{A} + (\text{grad}\phi \times \vec{A})$ **(07 Marks)**

OR

6 a. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $x^2 + y^2 - z = 3$ at $(2, -1, 2)$ **(06 Marks)**

b. Show that the vector field $\vec{F} = (x^2 - yz)\hat{i} + (y^2 - zx)\hat{j} + (z^2 - xy)\hat{k}$ is irrotational and find ϕ such that $\vec{F} = \nabla\phi$. **(07 Marks)**

c. Prove that $\text{div}(\phi\vec{A}) = \phi\,\text{div}\,\vec{A} + (\text{grad}\phi \cdot \vec{A})$ **(07 Marks)**

Module-4

7 a. Obtain the reduction formula for

$$\int_0^{\pi/2} \cos^n x\, dx$$ **(06 Marks)**

b. Solve $\frac{dy}{dx} = xy^3 - xy$ **(07 Marks)**

c. A body originally at 80°C cools down to 60°C in 20 minutes, the temperature of the air being 40°C. What will be the temperature of the body after 40 minutes from the original? **(07 Marks)**

OR

8 a. Evaluate $\int_0^{2a} \frac{x^2}{\sqrt{2ax - x^2}}\, dx$ **(06 Marks)**

b. Solve $(x^2 + y^2 + x)dx + xy\, dy = 0$ **(07 Marks)**

c. Obtain the orthogonal trajectories of the family of curves $r^n = a \sin n\theta$. **(07 Marks)**

Module-5

9 a. Find the rank of the matrix

$$A = \begin{bmatrix} 2 & 1 & 3 & 5 \\ 4 & 2 & 1 & 3 \\ 8 & 4 & 7 & 13 \\ 8 & 4 & -3 & -1 \end{bmatrix}$$ **(06 Marks)**

b. Diagonalize the matrix $\begin{bmatrix} -19 & 7 \\ -42 & 16 \end{bmatrix}$ **(07 Marks)**

c. Using Rayleigh's power method find the largest eigen value and the corresponding eigen vector of the matrix

$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ taking $(1\ 1\ 1)^T$ as the initial eigen vector. **(07 Marks)**

OR

10 a. Using Gauss-Siedel method, solve

$$20x + y - 2z = 17$$
$$3x + 20y - z = -18$$
$$2x - 3y + 20z = 25$$

using (0, 0, 0) as the initial approximation to the solution. **(06 Marks)**

b. Show that the linear transformation $y_1 = 2x_1 + x_2 + x_3$, $y_2 = x_1 + x_2 + 2x_3$, $y_3 = x_1 - 2x_3$ is regular and find the inverse transformation. **(07 Marks)**

c. Reduce the quadratic form $8x^2 + 7y^2 + 3z^2 - 12xy + 4xz - 8yz$ into the canonical form. **(07 Marks)**